Int. J. Heat Mass Transfer. Vol. 14, pp. 587–600. Pergamon Press 1971. Printed in Great Britain

GENERALIZED QUANTITATIVE CRITERIA FOR PREDICTING THE RATE-CONTROLLING MECHANISM FOR VAPOR BUBBLE GROWTH IN SUPERHEATED LIQUIDS

L. W. FLORSCHUETZ and A. S. AL-JUBOURI†

Mechanical Engineering Department, Arizona State University, Tempe, Arizona, U.S.A.

(*Received* 13 *April* 1970 *and in revised form* 28 *July* 1970)

Abstract—Criteria are presented for predicting the relative importance of the inertia of the liquid and heat transfer in the liquid as controlling effects for spherically symmetric vapor bubble growth in liquids. The coupled governing equations including both of the above effects are first cast in appropriate dimensionless forms. Then numerical solutions of the coupled equations are compared to the limiting solutions for solely liquid inertia and solely heat transfer controlled growth processes, in order to determine the respective regions of importance of these two mechanisms. The cases of growth initiated by a step decrease and a linear decrease in system pressure are both considered. The criteria are developed for the case of an idealized fluid having a constant vapor density and a linear vapor pressure curve, and then shown to be approximately independent of the vapor density and pressure relations of the particular fluid. The applicability and usefulness of the criteria are supported and illustrated by comparison of predictions based on them with previous theoretical and experimental results taken from the literature.

NOMENCLATURE

c_p, specific heat of liquid;
h_{fg}, enthalpy of vaporization;
Ja, Jakob number defined as $\rho c_p \Delta T/\rho_v(T_s)h_{fg}$;
k, thermal conductivity of liquid;
M, dimensionaless group used in [7]; equivalent to $4(R_n/R_0)^2$;
p_v, vapor pressure;
p_∞, instantaneous system pressure;
$p_{\infty,f}$, final system pressure;
Δp, pressure difference defined as $[p_v(T_\infty) - p_{\infty,f}]$;
R, bubble radius;
R_0, initial bubble radius;
R_d, bubble interface displacement, $(R - R_0)$;
R_n, characteristic length for normalization of bubble radius defined by $Ja^2 \kappa(\rho/\Delta p)^{\frac{1}{2}}$;
R^*, dimensionless bubble radius defined as $R/3\pi(\sqrt{3})R_n$;
R_0^*, dimensionless initial bubble radius defined as $R_0/3\pi(\sqrt{3})R_n$;
R_d^*, dimensionless bubble interface displacement defined as $R_d/3\pi(\sqrt{3})R_n$;
s, transformation variable defined as R^{*3};
T_i, bubble interface temperature;
T_s, saturation temperature corresponding to final system pressure;
T_∞, liquid temperature at large distance from bubble;
ΔT, superheat temperature difference defined as $(T_\infty - T_s)$;
t, time;
t_{HT}, time at which heat-transfer effects become dominant;
t_r, system pressure release time;

† Currently with College of Engineering, Baghdad University, Baghdad, Iraq.

t_n, characteristic time for normalization of bubble growth time defined by $Ja^2\,\kappa(\rho/\Delta p)$;
t^*, dimensionless time defined as $t/3\pi t_n$;
t_r^*, dimensionless system pressure release time defined as $t_r/3\pi t_n$;
u, transformation variable defined prior to equation (8).

Greek symbols
β, dimensionless group used in [7]; $\beta = 0$ corresponds to assumption of thermodynamic equilibrium at bubble interface;
ε_v, vapor pressure ratio defined as $\rho_v/\rho_v(T_s)$;
$\varepsilon_{v,\infty}$, initial vapor pressure ratio defined as $\rho_v(T_\infty)/\rho_v(T_s)$;
κ, thermal diffusivity;
π_v, dimensionless pressure ratio defined as $[p_v(T_\infty) - p_v(T_i)]/\Delta p$;
π_∞, dimensionless pressure ratio defined as $[p_v(T_\infty) - p_\infty(t)]/\Delta p$;
ρ, density of liquid;
ρ_v, density of vapor;
θ_i, dimensionless temperature ratio defined as $(T_\infty - T_i)/\Delta T$.

INTRODUCTION

A VAPOR bubble initially at equilibrium with its surrounding liquid will grow if the liquid pressure is decreased or if the temperature is increased. At early times the growth rate is controlled by the inertia of the liquid mass subject to the difference in pressure between the bubble interface and points in the liquid at a large distance from the interface. However, as the bubble grows, evaporation at the interface causes the temperature there to drop. The vapor pressure at the interface drops correspondingly until it reaches a value essentially equal to the liquid system pressure. The asymptotic growth is then controlled by the rate at which heat transfer from the liquid to the interface occurs in order to supply the necessary enthalpy of vaporization. For intermediate times, both liquid inertia and heat-transfer effects are significant in controlling the growth rate. Work on this coupled problem was first reported by Forster and Zuber [1] and by Plesset and Zwick [2]. They showed that for vapor bubbles growing from an equilibrium state in slightly superheated water at a pressure level of one atmosphere, the heat-transfer mechanism becomes controlling at such short times that liquid inertia effects may be entirely neglected. Earlier, Plesset [3] had shown that experimental radius–time curves for cavitation bubbles in water at subatmospheric pressure levels could be matched by considering only liquid inertia effects. In a subsequent review paper, Plesset [4] pointed out that liquid inertia effects become more significant as the pressure level is decreased. He noted that this was because the vapor density and the slope of the vapor pressure curve both decrease with the pressure level.

Since the early investigations mentioned above, the subject of vapor bubble growth has received much attention, most of it arising from interest in flashing, cavitation, and nucleate boiling phenomena. In spite of this, there does not appear to be available any practically usable criteria by which an investigator can obtain a good estimate of which growth controlling effects are important under his particular conditions. In addition to liquid inertia and heat transfer effects, other effects on the growth rate are surface tension, normal viscous stress at the bubble interface, and possible nonequilibrium effects at the interface. These effects have been included in analyses presented in several recent publications [6–8]. These papers present results for certain special cases involving particular fluids. While effects such as nonequilibrium may be important under some extreme conditions, it remains true that for macroscopic bubble growth under most conditions liquid inertia and/or heat-transfer effects are the dominant mechanisms. In order to put the relative importance of these effects on a more generalized quantitative basis, a theoretical investigation was performed using a single bubble model in an infinite body of superheated liquid with spherically symmetric phase growth. The coupled governing equations in appropriate dimensionless forms including both liquid inertia and heat-transfer effects were solved by numerical techniques resulting in radius–time curves. These solutions were compared to the solutions for the two limiting cases for purely heat transfer and purely liquid inertia controlled processes. In this way the respective regions where liquid inertia, heat transfer, or both effects control the growth rate were mapped out in terms of dimensionless quantities.

It is the purpose of this paper to report these results and to illustrate their validity and application. The latter is done by comparing experimental and theoretical results selected from the literature with the generalized results obtained here.

FORMULATION OF THE PROBLEM

The equation of motion for spherically symmetric bubble growth in an incompressible liquid is the Rayleigh equation,

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}(p_v - p_\infty). \qquad (1)†$$

Here the effects of surface tension and normal viscous stresses at the interface are neglected, and the condition $\rho_v \ll \rho$ must be satisfied.

We use two approximate solutions for the temperature at the bubble interface. One is a result due to Murdock [9] as reported by Bornhorst and Hatsopoulos [7],

$$T_\infty - T_i = \frac{(\kappa/2)^{\frac{1}{2}}\rho_v h_{fg}}{k}\left[\frac{(R^3 - R_0^3)\dot{R}}{R}\right]^{\frac{1}{2}}, \qquad (2)$$

obtained using an integral technique. The other is the well-known Plesset–Zwick solution [2] which may be written as

$$T_\infty - T_i = \frac{h_{fg}}{3k}\left(\frac{\kappa}{\pi}\right)^{\frac{1}{2}}\int_0^t \frac{\frac{d}{dt}(\rho_v R^3)}{\left[\int_z^t R^4(y)\,dy\right]^{\frac{1}{2}}}dz. \qquad (3)$$

† The dots denote time derivatives.

Both of these results are based on the assumption of a thin thermal boundary layer in the liquid adjacent to the bubble wall and constant liquid properties. Also, equation (2) involves the assumption that ρ_v is constant, while for equation (3) it is necessary only to assume that ρ_v is uniform. Equation (1) is coupled with equation (2) *or* equation (3) by the equilibrium vapor pressure and vapor density relations $p_v = p_v(T_i)$ and $\rho_v = \rho_v(T_i)$ assumed to be valid at the bubble interface. Initial conditions are $R = R_0$ and $\dot{R} = 0$ at $t = 0$. This completes the problem specification in dimensional form.

Normalization of the equations requires the selection of an appropriate characteristic length and time. One might select R_0 for the characteristic length, but this is not particularly appropriate for the case of bubble growth as it would be for collapse, since the asymptotic growth is independent of R_0. There is also no obvious characteristic time. However, normalization of the equations with respect to an arbitrary length and an arbitrary time, followed by inspection of the equations shows that selection of R_n and t_n as defined in the nomenclature is appropriate. With these, equations (1) and (2) become

$$R^*\ddot{R}^* + \tfrac{3}{2}\dot{R}^* = \tfrac{1}{3}[\pi_\infty - \pi_v] \qquad (4)$$

and

$$\theta_i = \left(\frac{9\pi}{2}\right)^{\frac{1}{2}}\left[\frac{(R^{*3} - R_0^{*3})\dot{R}^*}{R^*}\right]^{\frac{1}{2}}. \qquad (5)$$

These equations are coupled by

$$\pi_v = \pi_v(\theta_i) \text{ and } \varepsilon_v = \varepsilon_v(\theta_i), \qquad (6)$$

with initial conditions

$$R^* = R_0^*, \dot{R}^* = 0 \text{ at } t^* = 0. \qquad (7)$$

Normalization of equations (1) and (3) followed by transformation† according to

$$s = R^{*3} \text{ and } u = \int_0^{t^*} R^{*4}(y)\,dy,$$

†This transformation was originally used by Plesset and Zwick [2].

leads to

$$ss'' + \tfrac{7}{6}s'^2 = \frac{1}{s^{\frac{4}{3}}}[\pi_\infty - \pi_v] \qquad (8)†$$

and

$$\theta_i = \int_0^u \frac{(\varepsilon_v s)'}{(u - v)^{\frac{1}{2}}} \, dv \qquad (9)$$

with initial conditions

$$s = R_0^{*3}, s' = 0 \text{ at } u = 0. \qquad (10)$$

In summary, the equations to be solved are (4)–(7) when using Murdock's equation for the interface temperature and (8), (9), (6) and (10) when using the Plesset–Zwick equation for the interface temperature. We note that once $\pi_v(\theta_i)$, $\varepsilon_v(\theta_i)$, and the system pressure variation π_∞ are specified, these two sets of equations involve only one parameter, namely R_0^*.

For the sake of completeness, solutions for the limiting cases where either heat transfer or liquid inertia controls the entire bubble growth process are first outlined. For these cases the results in normalized form do not require the specification of a particular fluid. Next, the numerical solutions for the coupled case are examined. This requires selection of a particular fluid in order to specify $\pi_v(\theta_i)$ and $\varepsilon_v(\theta_i)$. We select two fluids: (i) an idealized fluid having a constant vapor density and a linear vapor pressure curve, and (ii) water. Results were obtained for two types of system pressure variation: (i) a step decrease, $\pi_\infty = 1$, for $t^* > 0$, and (ii) a linear decrease to a constant final value,

$$\left.\begin{aligned} \pi_\infty &= t^*/t_r^*, \quad 0 < t^* < t_r^* \\ \pi_\infty &= 1, \quad t^* > t_r{}^* \end{aligned}\right\} \qquad (11)$$

SOLUTIONS FOR LIMITING CASES

Liquid inertia controls

Here the vapor pressure is assumed to remain constant at its initial value, thus $\pi_v = 0$ in equation (4). For a step decrease in system pressure the solution to equation (4) is the well known Rayleigh solution. Graphical representations appear in Figs. 1, 5 and 6.

For a linear decrease in system pressure to a constant value, analytical solutions to equation (4) with $\pi_v = 0$, are not available. Numerical solutions have been obtained as part of this study. A graphical representation appears in Fig. 4.

Heat transfer controls

Neglecting the terms on the left hand side of equation (4) which represent liquid inertia effects leads to $\pi_v(\theta_i) = \pi_\infty$; i.e. the vapor pressure assumes a value always equal to the system pressure. Thus, for the step decrease in system pressure, the vapor pressure and hence the saturation temperature at the bubble interface remains constant during the growth, i.e. $\theta_i = 1$. Also, the vapor density remains constant, $\varepsilon_v = 1$. The solution to equation (5), the Murdock equation, is then

$$t^* = \frac{27\pi R_0^{*2}}{4}\left[\frac{1}{3}\left(\frac{R^*}{R_0^*}\right)^2 + \frac{2}{3}\left(\frac{R_0^*}{R^*}\right) - 1\right]. \qquad (12)$$

The solution to equation (9), the Plesset–Zwick equation, is

$$t^* = \frac{9\pi^2 R_0^{*2}}{4}\left[\frac{1}{3}\left(\frac{R^*}{R_0^*}\right)^2 + \frac{2}{3}\left(\frac{R_0^*}{R^*}\right) - 1\right]. \qquad (13)$$

As pointed out by Bornhorst and Hatsopoulus [7] these results, though based on different approximate techniques, give bubble radii differing by only about 2 per cent. Both approximations do, however, involve the thin thermal boundary layer assumption.

In examining the heat transfer limiting case for a linear decrease in system pressure to a constant value it is necessary to specify the vapor pressure relation. Assuming a linear vapor pressure relation, $\pi_v = \theta_i$, the temperature variation at the bubble interface will be linear for $0 < t^* < t_r^*$ and constant for $t^* > t_r^*$. The solution to equation (5) then becomes

$$t^* = \frac{81\pi t_r^* R_0^{*2}}{4}$$

†The primes denote derivatives with respect to u.

$$\left.\begin{array}{l} \left[\frac{1}{3}\left(\frac{R^*}{R_0^*}\right)^2 + \frac{2}{3}\left(\frac{R_0^*}{R^*}\right) - 1\right]^{1/3}, 0 < t^* < t_r^* \\ t^* = \frac{2}{3}t_r^* + \frac{27\pi R_0^*}{4} \\ \left[\frac{1}{3}\left(\frac{R^*}{R_0^*}\right)^2 + \frac{2}{3}\left(\frac{R_0^*}{R^*}\right) - 1\right], t^* > t_r^*. \end{array}\right\} \quad (14)$$

SOLUTIONS TO THE COUPLED EQUATIONS—IDEALIZED FLUID

The idealized fluid is taken to be one for which $\pi_v = \theta_i$ and $\varepsilon_v = 1$. Results are first presented for the step decrease and then for the linear decrease in system pressure.

Step decrease in system pressure

Equation (4) with $\pi_\infty = 1$ can be combined with equation (5) to give a single nonlinear ordinary differential equation with initial conditions given by equation (7). The only parameter is now R_0^*. This equation was integrated numerically by a fourth-order Runge–Kutta technique, for values of $R_0^* = 4$, 2, 1, 0·1, 0·01 and 0·001. A solution for one of these cases is displayed graphically in Fig. 1. The plot is for the dimensionless bubble interface displacement $R_d^* = R^* - R_0^*$ vs. t^*. For comparison the solutions for the corresponding limiting cases are displayed on the same plot. As would be expected, the solution for the early growth coincides with the limiting case for a liquid inertia controlled process, while the asymptotic growth eventually coincides with the solution for a heat transfer controlled process. The numerical solution was in agreement with equation (12) at times larger than those shown on the plot. For intermediate times the coupled solution clearly deviates from both limiting cases.

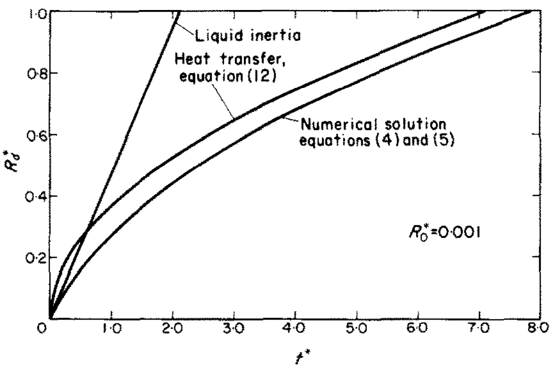

FIG. 1. Comparison between limiting and coupled solutions for idealized fluid with step pressure change.

In order to delineate those ranges of R_d^* and t^* over which a particular mechanism dominates, the following procedure was adopted. By comparing the numerical solutions for the coupled equations to the liquid inertia solution, values of R_d^* and t^* were selected at the point where R^* based on the liquid inertia solution deviates by 10 per cent from the value of R^* based on the coupled solution. Values were also chosen where the coupled solution for R^* comes to within 10 per cent of the value for the heat transfer solution. The selected values for R_d^* and t^* are plotted vs. R_0^*, (the only parameter) in Figs. 2 and 3 respectively. If one assumes that for the growth to be considered significant the radius must at least double, then the only region of interest on Fig. 2 is to the right of the line marked $R_d = R_0$. In this region the discriminating values of R_d^* are essentially independent of R_0^*. From Fig. 2 one may conclude that for $R_0^* > 0·3$ all significant growth is a

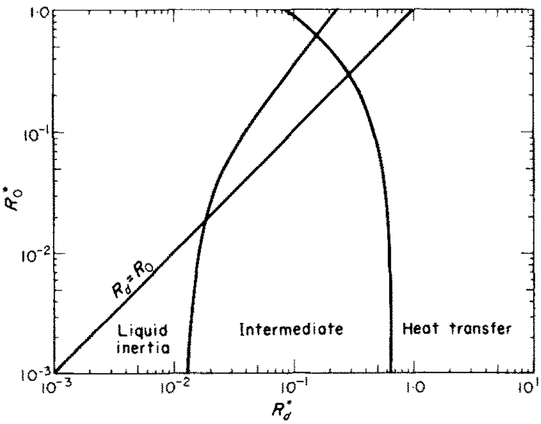

FIG. 2. Classification of growth controlling mechanism according to R_d^* as a function of R_0^*.

heat transfer controlled process. For $R_0^* < 0{\cdot}3$, a classification in terms of R_d^* must be made; i.e. for $R_d^* < 0{\cdot}013$ liquid inertia controls, for $R_d > 0{\cdot}66$ heat transfer controls, and $0{\cdot}013 < R_d^* < 0{\cdot}66$ is an intermediate region where both effects are significant.

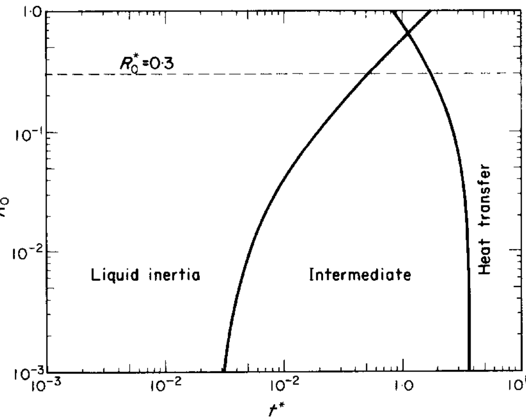

FIG. 3. Classification of growth controlling mechanism according to t^* as a function of R_0^*.

Turning to Fig. 3, and looking only at the region for $R_0^* < 0{\cdot}3$, the process may be said to be heat transfer controlled for $t^* > 3{\cdot}6$. The discriminating value of t^* for liquid inertia control does vary somewhat with R_0^*, but if one selects, say, $t^* < 0{\cdot}03$ liquid inertia control is assured. These results are summarized in Table 1. However, the summary has been put in terms of the dimensionless quantities R_0/R_n, R_d/R_n and t/t_n which differ only by constant factors from R_0^*, R_d^*, and t^* respectively.

Numerical solutions to the coupled equations (8) and (9), which involve the Plesset–Zwick approximation for the interface temperature, were also obtained for the idealized fluid case for values of R_0^* ranging from 0·05 to 4. These results, when compared to those using the Murdock approximation, agree for the early growth, deviate slightly for the intermediate growth, and eventually draw to within 2 per cent of each other for the asymptotic growth. The important point here is that the use of the solutions based on equations (8) and (9) for comparison to the limiting cases results in essentially the same criteria as is summarized in Table 1.

Linear decrease in system pressure

Here equations (4) and (5) were solved numerically with π_∞ given by equation (11) and the initial conditions given by equation (7). Now, in addition to R_0^* the pressure release time t_r^* is also a parameter. Since here the parametric effect of t_r^* is of major interest, solutions were obtained for six values of t_r^* ranging from 0·02 to 20, with $R_0^* = 0{\cdot}001$. The result for $t_r^* = 4$ is displayed in Fig. 4 and compared to the solutions for the corresponding limiting cases. The results for $t_r^* < 0{\cdot}03$ were essentially identical to those for the step change case, $t_r^* = 0$. For large values of t_r^*, the intermediate region starts to shrink and for $t_r^* > 4$ the heat-transfer

Table 1. Summary of approximate criteria for growth controlling mechanism—step pressure decrease

	Liquid inertia controls	Intermediate	Heat transfer controls
$R_0/R_n > 5$	—	—	Before radius doubles
$R_0/R_n < 5$	$R_d/R_n < 0{\cdot}2$	$0{\cdot}2 < R_d/R_n < 10$	$R_d/R_n > 10$
	$t/t_n < 0{\cdot}3$	$0{\cdot}3 < t/t_n < 30$	$t/t_n > 30$

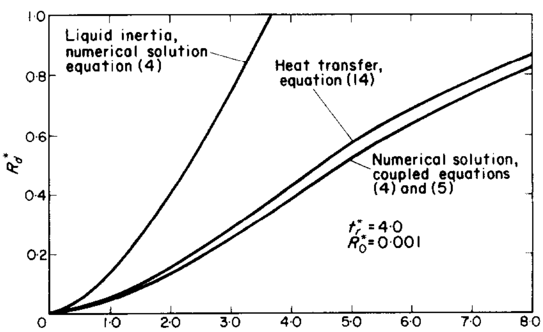

FIG. 4. Comparison between limiting and coupled solutions for idealized fluid with linear pressure variation to constant value at t_r^*.

limiting solution remains within 10 per cent of the coupled solution for the entire growth. For $0{\cdot}03 < t_r^* < 4$ the step change criteria can be used as a guide, but it should be kept in mind that the liquid inertia and intermediate regions will become less important as t_r^* increases. The suggested criteria for the linear system pressure change are summarized in Table 2, but have been put in terms of t_r/t_n rather than t_r^*.

Table 2. Summary of approximate criteria for growth controlling mechanism—linear pressure decrease for time t_r

$t_r/t_n < 0{\cdot}3$	Use criteria of Table 1
$0{\cdot}3 < t_r/t_n < 30$	Use criteria of Table 1, but note that liquid inertia and intermediate regions will shrink as t_r/t_n increases
$t_r/t_n > 30$	Heat transfer controls entire growth

Again, numerical solutions were also obtained using the coupled equations (8) and (9), with results essentially identical to those based on equations (4) and (5).

SOLUTIONS TO THE COUPLED EQUATIONS—REAL FLUID (WATER)

If the foregoing criteria based on idealized fluid results are to be of any practical use, their relationship to the case of real fluids must be examined. For a real fluid, the detailed growth curves, even in normalized form, will depend on the equilibrium vapor pressure and density relations for the particular fluid over the temperature range involved. But, the liquid inertia controlled growth (early) and the heat transfer controlled growth (asymptotic) do not depend on the shape of the vapor pressure and density relations of the fluid. Therefore, it seems possible that the points at which the true growth curve begins to deviate from the two limiting curves may not depend too strongly on these relations. To investigate this possibility, water was selected as a real fluid example. The Plesset–Zwick approximation in the form of equation (9) was used for the interface temperature since this result allows for a variable vapor density, whereas the Murdock approximation does not. Numerical solutions of the coupled equations (8) and (9), using empirical fits to actual water vapor pressure and density data for equation (6), were obtained for several temperature levels and degrees of superheat. Representative results are displayed in Figs. 5 and 6. For comparison, the results for the idealized fluid case are shown in the same figures. Comparison of Fig. 5 with Fig. 6 shows that for the same superheat, the smaller the temperature (or pressure) level, the larger the deviation of the real fluid result from the idealized fluid result. Results for the same temperature level but smaller superheats showed less deviation. These trends are to be expected.

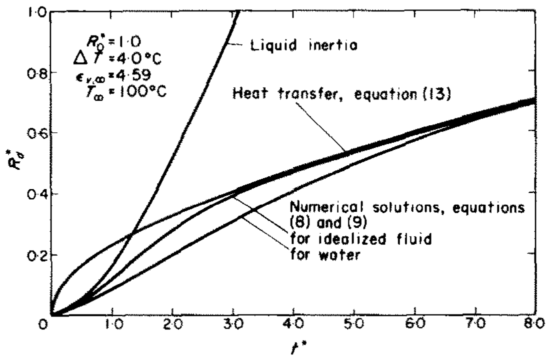

FIG. 5. Coupled solutions for idealized fluid and water compared to limiting cases with step pressure change.

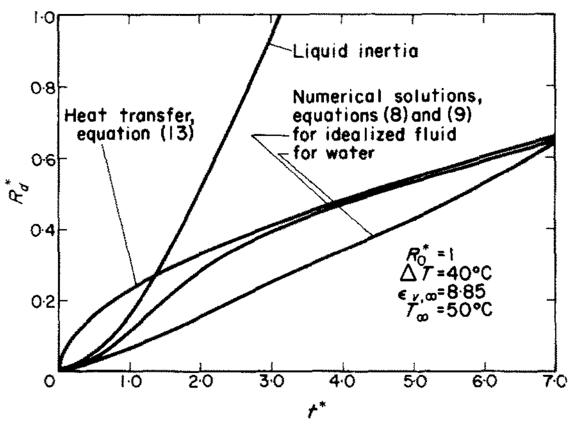

FIG. 6. Coupled solutions for idealized fluid and water compared to limiting cases with step pressure change.

The main aspect of these results is that the values of R_d^* and t^*, at which significant differences betweeen the solution curves for the coupled equations and the limiting cases are observed, differ by much less than an order of magnitude for the real fluid results as compared to the idealized fluid results. This is true even for the extreme case of Fig. 6, which has a low temperature level and a very large superheat resulting in an initial-to-final vapor density ratio of almost 9. It may be concluded that the criteria developed using the idealized fluid results will serve as a good approximate criteria for real fluids as well.

DISCUSSION

Griffith [5] in his early theoretical paper on bubble growth rates at a surface in boiling recognized that for large values of the Jakob number, the assumption that dynamic effects are unimportant is not valid. Cole and Shulman [11] pursued this point experimentally by obtaining data for bubble growth from a heated surface at subatmospheric pressures. They also related the relative importance of dynamic effects to the size of the Jakob number. While it is true that the relative importance of liquid inertia effects compared to heat transfer effects depends strongly on the Jakob number, that parameter is incomplete for the purpose at hand since it contains no information about the thermal conductivity or the slope of the vapor pressure curve of the liquid. In contrast, Tong [12] suggested that the measure of these effects can be expressed in the group $(R/\kappa)^2$ $(\Delta p/\rho)$. This group is also incomplete since it does not contain any information on the vapor density, the enthalpy of vaporization, or the slope of the vapor pressure curve. The dimensionless forms suggested here for bubble radius and growth time each contain all of the information contained separately in the two above-mentioned parameters. In fact, R/R_n is just the square root of the parameter mentioned by Tong divided by the square of the Jakob number. We note that Birkhoff, Margulies and Horning [13] suggested essentially that liquid inertia effects are negligible whenever $t \gg Ja^2\kappa/[8p(R)/\rho]$ or $R \gg Ja^2\kappa/[8p(R)/\rho]^{\frac{1}{2}}$, these criteria being very similar to those suggested here. However, no discriminating values were obtained and also, while these forms include the pressure level itself, they do not include what would seem to be more directly pertinent—a characterization of the slope of the vapor pressure curve which, of course, depends on the pressure level.

Effect of pressure level and superheat

For a given fluid, the relative importance of the two controlling mechanisms being considered here can depend quite strongly on the pressure level and the superheat. This is illustrated for water in Fig. 7. These results are based on the use of t/t_n reaching the value of 30 as indicated in Table 1 for heat transfer to become

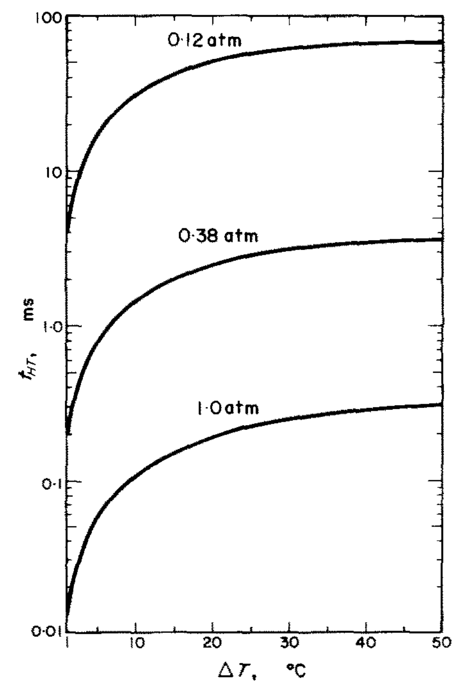

FIG. 7. Values of t_{HT} v. ΔT for water at several pressure levels.

the controlling mechanism. Note that for the smallest pressure level indicated (0·12 atm) liquid inertia effects will be significant since for $\Delta T > 3°C$ it requires 10 ms or longer to attain heat transfer controlled growth. This is mainly because as the pressure level is lowered, the vapor density and the slope of the vapor pressure curve both decrease. However, at one atmosphere and small superheats, this time is less than 0·1 ms. The conclusion that liquid inertia effects are not significant for macroscopic growth of water vapor bubbles at one atmosphere with small superheats was first reached by Plesset and Zwick [2] and Forster and Zuber [1] and more recently by Waldman and Houghton [6]. For experimental verification see Dergarabedian [14] and Florschuetz, Henry and Khan [15].

Perhaps more significantly, the present results show that this conclusion also holds for quite large superheats at pressure levels of one atmosphere and larger. A case in point is the experimental data obtained by Hooper and Abdelmessih [16] for spherically symmetric bubble growth under uniform superheats ranging from 6·8 to 38·8°C. For the smallest superheat the data was in good agreement with the theoretical curve based on a heat transfer model, but for the larger superheats the data fell increasingly below the theoretical curves. This discrepancy was tentatively attributed to the neglect of liquid inertia effects in the theoretical model. The present criteria indicate that even for their highest superheat this explanation is not correct. In particular, from Fig. 7, for 1 atmosphere and $\Delta T = 39°C$, one gets approximately 0·3 ms for attainment of heat transfer controlled growth, whereas Fig. 8 of [16] shows that the discrepancy exists for much larger times than this (data was obtained out to 3 ms). Thus, some other explanation must be sought. It has been suggested [10, 15] that although the growth was initiated by suddenly exposing the pressurized system to the atmosphere the pressure during bubble growth was still greater than one atmosphere and thus the actual equivalent superheats were less than those quoted in [16]. Theofonous, Biasi, Isbin and Fauske [8] suggested that the discrepancy may be attributed to interface non-equilibrium effects. Subsequent work by Hooper, Eidlitz and Faucher [17, 18] showed beyond doubt that the first explanation is the correct one.

Reference [17],† issued after the present study was completed, contains comprehensive data and analyses for over sixty tests of bubble growth in water at uniform (though not always constant) superheats. An "explicit correlation" was used to compare the growth data to a heat transfer model (termed asymptotic for constant superheats and quasi-asymptotic for time varying superheats), while an "implicit correlation" was used to compare the data to a model which included heat transfer, liquid inertia, and surface tension effects. Both correlations were based on use of the Plesset-Zwick result for the interface temperature, equation (3), but with the assumption of constant vapor density. Coincidence of the two correlations indicated asymptotic or quasi-asymptotic behavior, while significant deviation indicated nonasymptotic behavior. The importance of liquid inertia effects relative to heat transfer effects was also assessed by the use of a "reduced acceleration" basically defined as $\rho[R\ddot{R} + (3/2)\dot{R}^2]/[p_v(T_\infty) - p_\infty(t)]$, and evaluated using the experimental measurements for $R(t)$ and $p_\infty(t)$. When this quantity was less than 0·3 the growth was considered asymptotic, although for higher precision work use of the value 0·1 was suggested. The tests were classified into three main groups according to liquid temperature level with general conclusions regarding growth mode as follows: (a) high range, 280–200°F, the growth is truly quasi-asymptotic throughout; (b) intermediate range, 190–160°F, there is a gradual changeover from the truly quasi-asymptotic behavior of the high range to the nonasymptotic behavior of the low range; (c) low range, 140–100°F, the growth is highly nonasymptotic.

† The authors are indebted to Professor Hooper who kindly provided copies of [17] and an advance copy of [18].

Detailed application of the present criteria (Tables 1 and 2) to the conditions of these tests led to the same general conclusions. Several examples from each temperature range are summarized in Table 3. In each case $R_0/R_n < 5$.

Kosky [10] obtained data for water vapor bubbles growing at uniform superheats ranging from 10·5 to 36°C, and pressure levels from 0·477 to 1·19 atm. He obtained reasonable agreement with theoretical results which in-

Table 3. Present criteria compared with experimental results of [17]

Temp. range	Hooper, Eidlitz and Faucher [17]					Present criteria		Conclusions using bubble growth measurements [17]‖	
	Film No.	$T_\infty(°F)$ (T_0)†	$\Delta T(°F)$ $(T_0 - T_{opt})$†	t_r‡ (ms)	t_{obs}§ (ms)	t_r/t_n	t_{HT} (ms)	Reduced acceleration	Implicit vs. explicit correlations
High	31–8	240·8	13·3	1·5	16	1240	–	$<0·1, t > 0$	Coincide
	31–2	203·7	13·2	0·6	9	70	–	$<0·1, t > 0$	Coincide
Intermediate	31–6	184·0	16·7	0·7	4	17	$<1·2$	$\gtrsim 0·1, t > 0$	Very close
	30–17	181·4	9·5	0·6	7	32	–	$<0·3, >0·1, t < 1·2$ $<0·1, t > 1·2$	Coincide
	30–18	160·2	8·2	0·5	3	8·6	$<1·7$	$>0·3, t < 0·3$ $<0·3, t > 0·3$	Close–drawing together
	29–15	158·6	3·8	0·3	11	13	$<0·70$	$>0·3, t < 0·9$ $<0·3, t > 0·9$	Very close
Low	33–6	140·2	16·5	0·9	1	1·2	<22	$\gg 0·3, t > 0$	Way apart
	31–3	139·2	4·9	0·2	9	1·7	$<3·6$	$\gg 0·3, t < 1·6$ $<0·3, t > 1·6$	Apart—drawing together
	31–22	119·3	2·0	~0	11	$<0·3$	5·2	$\gg 0·3, t < 2·4$ $<0·3, t > 2·4$	Apart—coincide at 3·3 ms
	33–5	117·0	9·2	0·35	2	0·23	46	$\gg 0·3, t > 0$	Way apart
	32–1	101·6	4·1	~0	2	$<0·3$	50	$\gg 0·3, t > 0$	Way apart

† Notation of [17].
‡ Estimated from measured pressure curves given in [17].
§ Maximum valid or available bubble observation time.
‖ Based on graphical presentations given in [17].

Two of these cases, Films 31-3 and 31-22, are exceptions to the general conclusion for the low range, as was noted in [17]. Significantly, the present criteria also shows these two bubbles to be approaching asymptotic growth during the interval of observation. Note that this is mainly a result of the relatively low superheats for these cases. It is to be emphasized that the conclusions reached by Hooper *et al.* are based on a detailed examination of experimental results, while the present criteria are purely theoretical results. Their excellent extensive experimental results generally verify the validity of the present criteria.

cluded both liquid inertia and heat transfer effects [his equations (4) and (5)]. The present criteria applied to his conditions show that the time at which heat transfer effects become controlling ranges from about 0·1 ms for his Run 45 up to about 1 ms for his Run 58 (Bubble 2). Since the data for each of his runs covers a time interval from about 0·1 ms to 10 ms, most of it falls in the heat transfer region. Except for his Run 75, Kosky did not show a comparison to the asymptotic solution of Plesset and Zwick [Kosky's equation (5) with $T(R, t) = T_s$]. However, such a comparison, made by the present authors, shows that, except possibly for Run 58 (Bubble 2), the Plesset–Zwick theory matches the data as well as the theory curves computed by Kosky. In fact, if the Plesset–Zwick theory curves are plotted on Kosky's Figs. 9 and 10, they draw to within at least 10 per cent of Kosky's theoretical curves by the times predicted by the present criteria for heat transfer controlled growth. Although Kosky's data is valuable, bubble diameters at least as small as 0·01 cm would have to be measured to begin to clearly show liquid inertia effects for the conditions of his tests. This is an order of magnitude smaller than the smallest bubble sizes he apparently was able to measure with his experimental arrangement.

As an example of a case in which water vapor bubbles grew under conditions where liquid inertia effects dominated, the early work of Plesset [3] is cited. He numerically solved the modified Rayleigh equation incorporating only liquid inertia and surface tension effects and satisfactorily matched the radius–time data of Knapp and Hollander [19] for both the growth and collapse phases of cavitation bubbles on a streamlined body in a water tunnel. The growth times for these bubbles were less than 2 ms and for the pressure levels and Δp's involved result in values of t/t_n less than 0·25. Thus, the present criteria (Table 1) indicate that the entire growth should be in the liquid inertia region, which is consistent with Plesset's results.

Effect of fluid medium

For a given pressure level and equivalent superheat (or pressure difference), the relative importance of the two controlling mechanisms can depend quite strongly on the fluid medium involved. This is evident from Table 4, where the characteristic lengths and times are tabulated for three different fluids, all at the same pressure level and for the same superheat. It is clear that differences of several orders of magnitude exist. These values, in conjunction with Table 1, show that *for the given conditions*, liquid inertia effects are essentially negligible for the water, certainly so for the nitrogen, but may be significant for the potassium.

Table 4. Characteristic lengths and times for three fluids at atmospheric pressure and for $\Delta T = 10°C$

Fluid	R_n (cm)	t_n (s)
Water	$2{\cdot}2 \times 10^{-3}$	$3{\cdot}4 \times 10^{-6}$
Nitrogen	$1{\cdot}8 \times 10^{-4}$	$1{\cdot}1 \times 10^{-7}$
Potassium	$3{\cdot}8 \times 10^{-2}$	$9{\cdot}0 \times 10^{-5}$

An independent theoretical check on the present criteria can be obtained by comparing its predictions to theoretical curves presented by Bornhorst and Hatsopoulos [7] for nitrogen, water and potassium—their Figs. 3, 4 and 5 respectively. They plot $\dot{R}/\dot{R}_0$ vs. R/R_0 for the asymptotic solution and for the solution based on the coupled equations including heat transfer, liquid inertia, surface tension, and interfacial nonequilibrium effects. Of interest here are their asymptotic solution curves as compared to their solution curves for the coupled equations but neglecting non-equilibrium effects ($\beta = 0$). Noting that their M and our R_0/R_n are related by $R_0/R_n = 2/\sqrt{M}$ and using $R_d/R_n > 10$ for heat transfer controlled collapse from Table 1, leads to $R/R_0 = 51$ for nitrogen, their Fig. 3, 159 for water, their Fig. 4, and 5000 for potassium, their Fig. 5. These values are in excellent agreement with the points on the Bornhorst and Hatsopoulos curves where their coupled solution ($\beta = 0$) begins to coincide with their asymptotic solution ($\beta = 0$, $M = 0$).

Experimental verification that bubble growth in organic fluids for small, uniform superheats and atmospheric pressure levels is controlled by heat transfer has been reported by Dergarabedian [20] and by Florschuetz, Henry and Khan [15]. Similar verification for liquid nitrogen was reported by Hewitt and Parker [21] (see their Figs. 3–6).

Hewitt and Parker also obtained bubble diameter vs. time data (their Figs. 9–11) for liquid nitrogen subject to slow transient pressure release to one atmosphere. They compared the data to a numerical solution of equation (1) assuming p_v was constant and using the measured pressure variation for p_∞, and to the Plesset–

Zwick asymptotic solution. The data fell between these two solutions so a computer program for simultaneous solution of equations (1) and (3) was written, but results were not obtained because computer time requirements were too large. Subsequently, Theofonous *et al.* [8] obtained good agreement with the Hewitt and Parker data by obtaining numerical solutions for a theoretical model which included both heat transfer and liquid inertia effects, and also used the measured system pressure variation. The present criteria applied to liquid nitrogen at one atmosphere and $\Delta T = 10°C$ show that the time required to attain heat transfer controlled growth is of the order of 10^{-6} s. Hewitt and Parker's data were for ΔT's less than 10°C. It is, therefore, quite clear that a model based solely on heat transfer effects could be applied using the measured system pressure variation as essentially equivalent to the bubble interface pressure in order to fix the interface saturation temperature variation. Or, use of a reasonable value for t_r, which was about 0·1 s, results in $t_r/t_n \sim 10^6$. Comparison with Table 2 results in the same conclusion. Thus, retention of only heat transfer effects in the model of Theofonous *et al.* would give a theoretical curve identical to that of their Fig. 10, and also in satisfactory agreement with the data. Such a numerical solution would be considerably less involved than one for the coupled liquid inertia and heat-transfer equations. Indeed, if the system pressure variation can be approximated as linear, the analytical solution represented by equation (14) would be reasonably satisfactory.

In passing, it is also noted that the present criteria predict well the point at which Scriven's [22] asymptotic solution begins to coincide with the coupled solution (labeled equilibrium solution) as presented by Theofonous *et al.*, for one special case involving sodium (their Fig. 2) and one involving water (their Fig. 3).

Application to nonuniform superheat cases

Although the criteria developed here are based on a spherically symmetric model, it is of interest to make application to several cases of vapor bubbles growing from solid surfaces in nonuniformly superheated liquids such as occur during nucleate boiling processes.

Cole and Shulman [11] reported extensive diameter versus time data for bubbles growing from a horizontal ribbon in saturated nucleate pool boiling, with six different liquids at various pressure levels and wall superheats. They compared all of their data with the Forster–Zuber equation for spherically symmetric heat diffusion controlled growth [1], but with the superheat replaced by $(T_w - T_s)/2$ to take some account of the fact that the liquid superheat actually drops from $(T_w - T_s)$ at the wall to zero at some point away from the wall. Application of the present criteria to each of their tests showed (with the single exception of one toluene bubble) that whenever t_{HT} was of the same order or larger than the bubble observation time the data points fell significantly below the Forster–Zuber equation. But when t_{HT} was small compared to the observation time the agreement was fairly reasonable. This tends to verify the speculation given in [11] that liquid inertia effects were significant for some of their tests at subatmospheric pressures.

A very recent work by Sernas and Hooper [23] had as a major objective the determination of t_{HT} based on experimental results. A result was reported for water vapor bubbles growing at a heated wall during saturated nucleate boiling at a pressure level of one atmosphere for $T_w - T_s = 23°F$ (12·8°C). The estimated initial growth period from nucleation to the time at which growth begins to follow the law $R =$ (constant) $t^{\frac{1}{2}}$ was determined to be in the range 43–60 μs, with 50 μs as the most probable value. The zero time datum was established by the use of streak photography in combination with framing photography. If, to account somewhat for the nonuniform superheat adjacent to the heating surface, one uses an average value $(T_w - T_s)/2$ for the value of ΔT, Fig 7 gives t_{HT} about 70 μs. The agreement with the experimental value reported by [23] is excellent, particularly when one considers the relationship between the model on which the prediction is based and the actual experimental conditions.

CONCLUDING REMARKS

The authors believe that the particular dimensionless variables used here are the most appropriate that have been suggested to date for determination of the relative importance of liquid inertia and heat transfer effects. Furthermore, particular discriminating values of these dimensionless variables have been suggested as summarized in Tables 1 and 2. That these criteria lead to reasonable predictions is supported by the comparisons with previous experimental and theoretical work as outlined in the discussion presented in the preceding section. Estimates based on the criteria should be useful to investigators in designing experiments involving vapor bubble growth phenomena and in selecting or developing appropriate theoretical models for comparison with the data. If resulting estimates are used with proper caution, the criteria can also be applied to bubble growth phenomena associated with boiling heat transfer and flashing or cavitation processes occurring in practice under various particular conditions.

ACKNOWLEDGEMENTS

This work was supported in part by the National Science Foundation under Grant No. GK-229. Computer time was furnished at no charge by the Arizona State University Computer Center.

REFERENCES

1. H. K. Forster and N. Zuber, Growth of a vapor bubble in superheated liquid, *J. Appl. Phys.* **25**, 474–478 (1954).
2. M. S. Plesset and S. A. Zwick, The growth of vapor bubbles in superheated liquids, *J. Appl. Phys.* **25**, 493–500 (1954).
3. M. S. Plesset, The dynamics of cavitation bubbles, *J. Appl. Phys.* **16**, 277–282 (1949).
4. M. S. Plesset, Physical effects in cavitation and boiling, *Symposium on Naval Hydrodynamics*, National Academy of Science, National Research Council, 297–317 (September 1956).
5. P. Griffith, Bubble growth rates in boiling, *Trans. Am. Soc. Mech. Engrs* **80**, 721–726 (1958).
6. L. A. Waldman and G. Houghton, Spherical phase growth in superheated liquids, *Chem. Engng Sci.* **20**, 625–636 (1965).
7. W. G. Bornhorst and G. N. Hatsopoulos, Bubble growth calculation without neglect of interfacial discontinuities, *J. Appl. Mech.* **34**, 847–853 (1967).
8. T. Theofonous, L. Biasi, H. S. Isbin and H. Fauske, A theoretical study on bubble growth in constant and time-dependent pressure fields, *Chem. Engng Sci.* **24**, 885–897 (1969).
9. J. Murdock, Internal report in Mechanical Engineering Department, Massachusetts Institute of Technology (1965).
10. P. G. Kosky, Bubble growth measurements in uniformly superheated liquids, *Chem. Engng Sci.* **23**, 695–706 (1968).
11. R. Cole and H. L. Shulman, Bubble growth rates at high Jakob numbers, *Int. J. Heat Mass Transfer* **9**, 1377–1390 (1966).
12. L. S. Tong, *Boiling Heat Transfer and Two-Phase Flow*, p. 14. John Wiley, New York (1965).
13. G. Birkhoff, R. S. Margulies and W. A. Horning, Spherical bubble growth, *Physics Fluids* **1**, 201–204 (1958).
14. P. Dergarabedian, The rate of growth of vapor bubbles in superheated water, *J. Appl. Mech.* **20**, 537–545 (1953).
15. L. W. Florschuetz, C. L. Henry and A. Rashid Khan, Growth rates of free vapor bubbles in liquids at uniform superheats under normal and zero gravity conditions, *Int. J. Heat Mass Transfer* **12**, 1465–1489 (1969).
16. F. C. Hooper and A. H. Abdelmessih, The flashing of liquids at higher superheats, *Proc. Third Int. Heat Transfer Conf.* **4**, 44–50 (1966).
17. F. C. Hooper, A. Eidlitz and G. Faucher, *Bubble Growth and Pressure Relationships in the Flashing of Superheated Water*, Vols. 1, 2 and 3, UTME-TP 6904, University of Toronto, Dept. of Mech. Engng (July 1969)
18. F. C. Hooper, A. Eidlitz and G. Faucher, Pressure effects on bubble growth in the flashing of superheated water, to be presented at the 4th Int. Heat Transfer Conference, Paris, France, Aug. 31–Sept. 4 (1970).
19. R. T. Knapp and A. Hollander, Laboratory investigations of mechanism of cavitation, *Trans. Am. Soc. Mech. Engrs* **70**, 419 (1948).
20. P. Dergarabedian, Observations on bubble growth in various superheated liquids, *J. Fluid Mech.* **9**, 39–48 (1960).
21. H. C. Hewitt and J. D. Parker, Bubble growth and collapse in liquid nitrogen, *J. Heat Transfer* **90**, 22–26 (1968).
22. L. E. Scriven, On the dynamics of phase growth, *Chem. Engng Sci.* **1**, 1–13 (1959).
23. V. Sernas and F. C. Hooper, The initial vapor bubble growth on a heated wall during nucleate boiling, *Int. J. Heat Mass Transfer* **12**, 1627–1639 (1969).

CRITÈRES QUANTITATIFS GÉNÉRAUX POUR PRÉDIRE LE MÉCANISME CONTRÔLANT LA CROISSANCE DE BULLES DE VAPEUR DANS DES LIQUIDES SURCHAUFFÉS

Résumé—On présente des critères afin de prédire l'importance relative de l'inertie du liquide et le transfert thermique dans le liquide, comme contrôlant la croissance d'une bulle de vapeur à symétrie sphérique dans des liquides. Le système d'équations renfermant les deux effets ci-dessus est présenté sous une forme adimensionelle appropriée. Les solutions numériques du système d'équations sont comparées aux solutions limites du processus de croissance contrôlé uniquement par inertie du liquide et uniquement par le transfert thermique afin de déterminer les régions d'importance respective de ces deux mécanismes. On considère les cas de croissance initiée par une décroissance échelon et une décroissance linéaire de la pression du système. Les critères sont développés dans le cas d'un fluide idéalisé ayant une densité de vapeur constante, et une courbe de pression de vapeur linéaire et on montre ensuite qu'ils sont approximativement indépendants des lois de densité de vapeur et de pression de ce fluide particulier. L'applicabilité et l'utilité de ces critères sont établies et illustrées par comparaison des prédictions basées sur eux aux études théoriques et aux résultats expérimentaux pris dans la littérature.

VERALLGEMEINERTE QUANTITATIVE KRITERIEN FÜR VORAUSSAGEN ÜBER DEN REGELMECHANISMUS VON BLASENWACHSTUMSRATEN IN ÜBERHITZTEN FLÜSSIGKEITEN

Zusammenfassung—Für Voraussagen über die relative Bedeutung der Massenträgheit und des Wärmeüberganges in der Flüssigkeit als regelnde Einflüsse auf das kugelsymmetrische Blasenwachstum in Flüssigkeiten werden Kriterien angegeben. Die gekoppelten regulierenden Gleichungen sind unter Berücksichtigung der oben angeführten Effekte zuerst in passende dimensionslose Formen gebracht worden. Die numerischen Lösungen der gekoppelten Gleichungen werden dann mit den einschränkenden Lösungen für nur trägheitskraftgeregelte und nur wärmeübergangsgeregelte Wachstumsprozesse verglichen, um die betreffenden Bereiche nach der Wichtigkeit dieser beiden Mechanismen zu beurteilen. Beide Fälle des Blasenwachstums, ob sie durch stufenförmige, oder lineare Druckabnahme im System eingeleitet wurden, sind berücksichtigt worden. Die Kriterien werden für den Fall einer idealen Flüssigkeit mit konstanter Dampfdichte und einer linearen Dampfdruckkurve entwickelt. Es zeigt sich dabei, dass die Kriterien annähernd unabhängig von der Dampfdichte und den Druckbeziehungen der einzelnen Flüssigkeiten sind. Die Anwendbarkeit und Nützlichkeit der Kriterien werden unterstützt und veranschaulicht durch den Vergleich der auf diesen Kriterien beruhenden Voraussagen mit früheren theoretischen und experimentellen Ergebnissen aus der Literatur.

ОБОБЩЕННЫЕ КОЛИЧЕСТВЕННЫЕ КРИТЕРИИ ДЛЯ РАСЧЕТА МЕХАНИЗМА РОСТА ПУЗЫРЬКОВ ПАРА, ЗАВИСЯЩЕГО ОТ ЕГО ИНТЕНСИВНОСТИ, В ПЕРЕГРЕТЫХ ЖИДКОСТЯХ

Аннотация—Представлены критерии для расчета относительного влияния инерции жидкости и теплообмена в жидкости как определяющих механизмов при росте сферически симметричных пузырьков пара в жидкостях. Система определяющих уравнений, описывающих влияние обоих названных механизмов, дана сначала в безразмерном виде. Затем следует сравнение численных решений системы уравнений с решениями для крайних случаев отдельно для инерции жидкости и отдельно для теплопереноса как механизмов, определяющих процессы роста, с целью определения наиболее важных участков двух механизмов. Рассматриваются случаи роста, вызванного ступенчатым и линейным уменьшениями давления в системе. Разработаны критерии для случая идеализированной жидкости с постоянной плотностью пара и линейной зависимостью давления пара, которые далее рассматриваются приблизительно независимыми от отношения плотности пара и отношения давления в этой жидкости. Чтобы продемонстрировать применимость данных критериев, проводится сравнение расчетов, проведенных по этим критериям, с известными в литературе теоретическими и экспериментальными данными.